Lyapunov-Function-Based Event-Triggered Control of Nonlinear Discrete-Time Cyber–Physical Systems

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***Abstract*—In this brief, a new Lyapunov-function-based event-triggered mechanism is proposed to investigate the state feedback control of nonlinear discrete-time cyber-physical systems. By summing the difference of Lyapunov-function between two adjacent events and utilizing it to construct the triggering law, the asymptotic stability of system can be ensured without demanding the difference of the Lyapunov function to be negative at any time. Thus, a larger inter-execution time under this scheme can be obtained than the conventional event-triggered mechanism. Meanwhile, the application of this scheme to linear systems is also presented. Finally, some simulations are implemented to show the advantages of the developed strategy.**

***Index Terms*—Event-triggered control, cyber-physical systems, networked control systems, Lyapunov function.**

I. Introduction

In CYBER-PHYSICAL systems (CPSs), many communication channels are usually realized via shared wire/wireless networks to exchange data among system components [1]–[3]. Nowadays, most CPSs operate based on a periodic communication technique, which is also called time-driven scheme. This manner may be conservative since it is difficult to take a trade-off between limited network resource and prescribed control performance. Compared with the conventional time-driven scheme, an event-triggered mechanism (ETM) has attracted increasing attention for its potential ability of reducing the frequency of data transmission [4]–[7].

The main feature of ETM is that the control task is implemented only when a predesigned triggering rule, that is related with the system state, is violated. Its effectiveness of decreasing the updating frequency of control task is verified by some experimental results in [5], where the network resources are saved dramatically while the desired stability performance is also maintained. Regarding this topic, fruitful theoretic results have been reported in [6], [7] and the references therein. In [6], a state-dependent event-triggered strategy, called as a static ETM, is proposed to ensure the input-to-state stability (ISS). Note that this brief requires a monotonic Lyapunov function (LF), which has been proven that a small increase can be endured by using an exponential stability method in [7]. Following this idea, various kinds of ETMs have been investigated for linear and nonlinear networked control systems. To be specific, in [8], a dynamic ETM is presented by introducing an internal variable into the static ETM to ensure the system stability. Based on this treatment, the term $\sigma\alpha(\|x(t)\|)-\gamma(\|e(t)\|)$ used to trigger events in the static ETM is not required to be negative at all time. An integral-based ETM related with the integral of LF is studied for linear and nonlinear systems in [9] and [10]. This ETM is extended to linear system with bounded disturbances in [11].

The approaches in previously discussed outcomes are analyzed in a continuous-time case. The event-triggered control of discrete-time nonlinear systems under the ISS frame is addressed in [12], where the term $\sigma\alpha(\|x(k)\|)-\gamma(\|e(k)\|)$ used to trigger events is always required to be less than zero. In [13], the nonlinear small-gain theorem is utilized to deal with the event-triggered control issue of nonlinear discrete-time systems with disturbances. For discrete-time neural networks subject to unknown delays, [14] studies the event-triggered synchronization problem. In these results, however, the chosen LFs are monotonically decreasing under the considered ETMs.

Motivated by the above observations, a novel LF-based ETM is presented for nonlinear discrete-time CPSs in this brief. With the introduction of the sum of LF between two consecutive events, the triggering condition can be relaxed and the LF allows little increases. Then, the inter-execution time under the proposed LF-based ETM is proved to be larger than the traditional ETM in [12].

Notation: In this brief, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ stand for the n dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\|\cdot\|$ is used to represent the Euclidean norm of a vector or matrix. $X^\top$ means the transpose of a vector or matrix X.

II. Preliminaries

Consider the following nonlinear system as

$$x(k+1)=h(x(k),u(k)),\ \ x(k)\in\mathbb{R}^n,\ \ u(k)\in\mathbb{R}^m, \qquad (1)$$

where $h:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ is assumed to be continuous with initial condition $h(0,0)=0$. Namely, the origin is an equilibrium point for system (1). With the utilization of zero-order

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holder (ZOH), the actual control of (1) between two adjacent triggering instants is given by

$$u(k) = g(x(k_i)),\ k \in [k_i,\ k_{i+1}) \triangleq \Omega_i, \tag{2}$$

where $g : \mathbb{R}^n \to \mathbb{R}^m$ is continuous control law, the elements of the increasing sequence $\mathbb{T} = \{k_1, k_2, \dots, k_\infty\}$ are the triggering instants generated by the normal ETM (4) or the LF-based ETM (6) given as follows.

Defining the error vector $\epsilon(k) = x(k_i) - x(k)$, then, the closed-loop system is established as:

$$x(k+1) = h(x(k) + g(x(k) + \epsilon(k))),\ k \in \Omega \triangleq \cup\Omega_i. \tag{3}$$

A. Normal ETM

To compare with the LF-based ETM designed later, the normal ETM proposed in [12] is provided first as

$$k_{i+1} = \inf_{k\in\mathbb{N}}\{k > k_i | s(x(k), \epsilon(k)) > 0\}, \tag{4}$$

where $\delta \in (0,\ 1)$, $s(x(k), \epsilon(k)) = \xi(\|\epsilon(k)\|) - \delta\zeta(\|x(k)\|)$.

It is seen that combining this ETM and the condition (8), one has

$$\Delta V(x(k)) \le (\delta - 1)\zeta(\|x(k)\|) < 0, \tag{5}$$

for $\delta \in (0,\ 1)$, which ensures that the state of system is stabilized to the origin asymptotically.

B. LF-Based ETM

By adding an LF-based term into the normal ETM (4), a novel LF-based ETM is constructed as

$$k_{i+1} = \inf_{k\in\mathbb{N}}\{k > k_i | s(x(k), \epsilon(k)) + \theta \sum_{\kappa=k_i}^{k} \Delta V(x(\kappa)) > 0\}, \tag{6}$$

where $\theta > 0$.

Remark 1: With the introduction of $\theta \sum_{\kappa=k_i}^{k} \Delta V(x(\kappa))$, the normal ETM $s(x(k), \epsilon(k))$ is not required to be negative at all time. Thus, the proposed LF-based ETM could generate a larger inter-event time than the normal ETM, the strict proof of which is provided in Proposition 1. Moreover, as the increase of the scalar θ, the triggering condition $k_{i+1} = k_i + \inf_{k\in\mathbb{N}}\{k > k_i | s(x(k), \epsilon(k)) + \theta \sum_{\kappa=k_i}^{k} \Delta V(x(\kappa)) > 0\}$ becomes more difficult to be violated. This means that if it requires save more network resources, a larger θ can be chosen. On the contrary, a smaller θ can be chosen to trigger more control signals and obtain better control performance.

Next, the definition of discrete-time LF and a useful lemma are given before deriving the stability theorem under the LF-based ETM (6).

Definition 1: A continuous function $V : \mathbb{R}^n \to \mathbb{R}^+$ is an ISS-LF for system (3), if there exist $\mathcal{K}_\infty$ functions ζ_1, ζ_2, ζ, and $\mathcal{K}$ function ξ such that for all $x(k)$ and $\epsilon(k) \in \mathbb{R}^n$,

$$\zeta_1(\|x(k)\|) \le V(x(k)) \le \zeta_2(\|x(k)\|), \tag{7}$$

$$\Delta V(x(k)) \le -\zeta(\|x(k)\|) + \xi(\|\epsilon(k)\|), \tag{8}$$

where $\Delta V(x(k)) \triangleq V(x(k+1)) - V(x(k))$.

Lemma 1: If there exists an LF satisfying Definition 1 such that

$$V(x(k_{i+1})) - V(x(k_i)) = \sum_{\kappa=k_i}^{k_{i+1}-1} \Delta V(x(\kappa)) < 0, \tag{9}$$

$$V(x(k)) < V(x(k_i)), \quad \forall k \in \Omega_i, \tag{10}$$

then the event-triggered control system (3) is asymptotically stabilized to the origin.

Proof: In terms of (7), (9) and (10), it yields

$$\zeta_1(\|x(k)\|) \le \zeta_2(\|x(k_1)\|), \quad \forall k > k_1, \tag{11}$$

where $x(k_1)$ is initial condition of system. Therefore, for any $x(k_1)$ satisfying $\|x(k_1)\| \le \eta_V = \zeta_2^{-1}(\zeta_1(\alpha_V))$, we obtain

$$\|x(k)\| \le \alpha_V, \quad \forall k > k_1, \tag{12}$$

for a given $\alpha_V > 0$, which means the system is stable.

From (10), there exists a positive scalar γ_i such that

$$\mathbb{I}_i = \{x(k) | V(x(k)) + \gamma_i \le V(x(k_i))\}, \quad k_i \in \mathbb{T} \tag{13}$$

is an invariant set including the origin. In addition, it is resulted from (10) that $\mathbb{I}_{i+1} \subset \mathbb{I}_i$, $\forall i \in \mathbb{N}$. It gives that $\mathbb{I}_i$ makes up a shrinking sequence of invariant sets, represented by $\mathbb{I} = \{\mathbb{I}_i : \forall i \in \mathbb{N}\}$. To guarantee the system state to be asymptotically convergent, it equals to prove

$$\mathbb{I}_i \to \mathbb{O}, \quad i \to \infty, \tag{14}$$

where $\mathbb{O} = \{0\}$ means the singleton set only including the origin.

It is supposed that $\mathbb{O}$ includes at least a state $y \ne 0$ and

$$V(x(k_i)) \to V(y) > 0, \quad i \to \infty. \tag{15}$$

Take into account the following two compact sets

$$\Gamma_0 = \{x(k) | V(x(k)) \le V(x(k_1))\}, \tag{16}$$

$$\Gamma_1 = \{x(k) | V(x(k)) \le V(y)\}, \tag{17}$$

and define two positive scalars r and d meeting

$$\Gamma_0 \subset \mathbb{B}_r = \{x(k) | \|x(k)\| \le r\}, \tag{18}$$

$$\Gamma_1 = \{x(k) | \|x(k)\| \le d\} \subset \mathbb{B}_d. \tag{19}$$

According to (15), one knows that $x(k_i)$ belongs to $\mathbb{B}_r$ but exceeds $\mathbb{B}_d$ as $\forall i \in \mathbb{N}$:

$$d \le \|x(k_i)\| \le r, \quad i = 1, 2, \dots \tag{20}$$

By letting $\rho = \max_{d \le \|x(k_i)\| \le r}\{V(x(k_{i+1}) - V(x(k_i))\}$ and from (9), one can derive

$$V(x(k_{i+1})) \le V(x(k_1)) + i\rho. \tag{21}$$

Due to $\rho < 0$, there exist some l for $i \ge l$ such that $V(x(k_{i+1})) < 0$, which contradicts $V(x(k)) > 0$. Therefore, $V(x(k_i)) \to 0$ as $i \to \infty$, which guarantees that $x(k)$ converges asymptotically to the origin. ■

Remark 2: For a chosen ISS-LF, if $\Delta V(x(k)) \le -\zeta(\|x(k)\|) + \xi(\|\epsilon(k)\|) < 0$, the closed-loop system (3) is said to be input-to-state stable. If a chosen ISS-LF satisfies the conditions in Lemma 1, the closed-loop system (3) is asymptotically stable. In addition, if there exists a positive constant α such that $\|x(k)\| \le \alpha$, the closed-loop system (3) is said to be stable in Lyapunov sense.

III. Main Results

In this part, the stability of system (3) using the LF-based ETM (6) is produced in Theorem 1. A new proposition is derived to show that a larger inter-event time is generated via the LF-based ETM (6) than the normal ETM (4).

Theorem 1: For given continuous Lipschitz functions h, g, ζ and ξ, the closed-loop system (3) under the LF-based ETM (6) is asymptotically stable, if there exists an LF satisfying Definition 1 such that (9) and (10) hold.

Proof: An LF $V(x(k))$ satisfying the conditions in Definition 1 is selected.

From the definition of $\Delta V(x(k))$, it yields

$$\sum_{\kappa=k_i}^{k} \Delta V(x(\kappa)) = V(x(k+1)) - V(x(k_i)). \tag{22}$$

For $k \in \Omega_i$, by combining (22) and the triggering condition (6), it yields

$$s(x(k), \epsilon(k)) + \theta(V(x(k+1)) - V(x(k_i))) \leq 0. \tag{23}$$

According to (8) and (23), we have

$$\begin{aligned} s(x(k), \epsilon(k)) &+ \theta(V(x(k+1)) - V(x(k_i))) + \Delta V(x(k)) \\ &\leq -\zeta(\|x(k)\|) + \xi(\|\epsilon(k)\|), \end{aligned} \tag{24}$$

which equals to

$$\begin{aligned} (1+\theta)V(x(k{+}1)) &- \theta V(x(k_i)) - V(x(k)) \\ &\leq (\delta - 1)\zeta(\|x(k)\|). \end{aligned} \tag{25}$$

From (25) and $\delta \in (0, 1)$, one can get

$$\begin{aligned} V(x(k+1)) &\leq \frac{1}{1+\theta}V(x(k)) + \frac{\theta}{1+\theta}V(x(k_i)) \\ &\quad + \frac{\delta-1}{1+\theta}\zeta(\|x(k)\|) \\ &< \frac{1}{1+\theta}V(x(k)) + \frac{\theta}{1+\theta}V(x(k_i)). \end{aligned} \tag{26}$$

In terms of the comparison lemma and $\theta > 0$, it gives

$$\begin{aligned} V(x(k)) &< \left(\frac{1}{(1+\theta)^{k-k_i}} + \sum_{j=1}^{k-k_i} \frac{\theta}{(1+\theta)^j}\right) V(x(k_i)) \\ &= V(x(k_i)). \end{aligned} \tag{27}$$

From (27), one can get $V(x(k_{i+1})) < V(x(k_i))$.

By applying Lemma 1, the closed-loop event-triggered control system (3) is asymptotically stable. ■

Remark 3: It is seen that the system stability under the LF-based ETM is guaranteed by $V(x(k)) - V(x(k_i)) < 0$, instead of the difference $V(x(k+1)) - V(x(k)) < 0$ under the normal ETM. In this case, the chosen LF allows some little increases and is not strictly monotonic decreasing.

Next, we provide a proposition to demonstrate that the proposed LF-based ETM (6) could generate larger inter-event time than the normal ETM (4).

Proposition 1: Considering the stabilized system (3), let $k_i = k_i^s = k_i^d$, k_{i+1}^s be given by normal ETM (4) and k_{i+1}^d be given by LF-based ETM (6), then $k_{i+1}^s \leq k_{i+1}^d$.

Proof: By assuming that $k_{i+1}^s > k_{i+1}^d$ and substituting k_{i+1}^d into (4), it gives

$$s(x(k_{i+1}^d), \epsilon(k_{i+1}^d)) = \xi(\|\epsilon(k_{i+1}^d)\|) - \delta\zeta(\|x(k_{i+1}^d)\|) < 0. \tag{28}$$

According to (6), one has

$$s(x(k_{i+1}^d), \epsilon(k_{i+1}^d)) = \theta V(x(k_i)) - \theta V(x(k_{i+1}^d)). \tag{29}$$

From the stabilized system (3) and Lemma 1, it yields

$$V(x(k_i)) > V(x(k_{i+1}^d)). \tag{30}$$

Then, combining (29) and (30), one can get

$$\xi(\|\epsilon(k_{i+1}^d)\|) - \delta\zeta(\|x(k_{i+1}^d)\|) > 0, \tag{31}$$

which contracts $s(x(k_{i+1}^d), \epsilon(k_{i+1}^d)) < 0$ in (28). Hence, $k_{i+1}^s \leq k_{i+1}^d$ is derived. ■

IV. Application to Linear System Case

In this section, we discuss the control system in linear case of the form

$$x(k+1) = Ax(k) + Bu(k). \tag{32}$$

With the event-triggered controller $u(k) = Kx(k_i)$, $k \in \Omega_i$, the event-triggered control system is established as:

$$x(k+1) = (A + BK)x(k) + BK\epsilon(k). \tag{33}$$

Thus, there exists an ISS-LF $V(x(k)) \triangleq x^\top(k)Px(k)$ for system (32). The matrix $P > 0$ satisfies

$$(A + BK)^\top P(A + BK) - P = -Q, \tag{34}$$

in which $Q > 0$ can be any symmetric matrix.

The normal ETM for discrete linear systems is given as:

$$k_{i+1} = k_i + \inf_{k\in\mathbb{N}}\{k > k_i | \hat{s}(x(k), \epsilon(k)) > 0\}, \tag{35}$$

in which $\delta \in (0, 1)$,

$$\begin{aligned} \hat{s}(x(k), \epsilon(k)) = \delta x^\top(k)Qx(k) &- \epsilon^\top(k)K^\top B^\top PBK\epsilon(k) \\ &- 2x^\top(k)(A + BK)^\top PBK\epsilon(k). \end{aligned}$$

Remark 4: If the assumption $k^* > 1$ in [12, Proposition 2] holds, we have the inter-event time $\tau_{\min} = k_{i+1}^s - k_i > 1$. Then, $k_{i+1}^d - k_i > 1$ under the LF-base ETM is obtained. This means the LF-base ETM (6) needs at least two steps for the next controller update.

In the sequel, our proposed LF-based ETM for linear systems is designed as:

$$k_{i+1} = \inf_{k\in\mathbb{N}}\{k > k_i | \hat{s}(x(k), \epsilon(k)) + \theta \sum_{\kappa=k_i}^{k} \Delta V(x(\kappa)) > 0\}, \tag{36}$$

where $\delta \in (0,\ 1)$, $\theta > 0$.

In terms of Proposition 1, one can easily get $k_{i+1}^d - k_i \geq k_{i+1}^s - k_i$. Then a larger inter-event time is obtained by LF-based ETM than normal ETM.

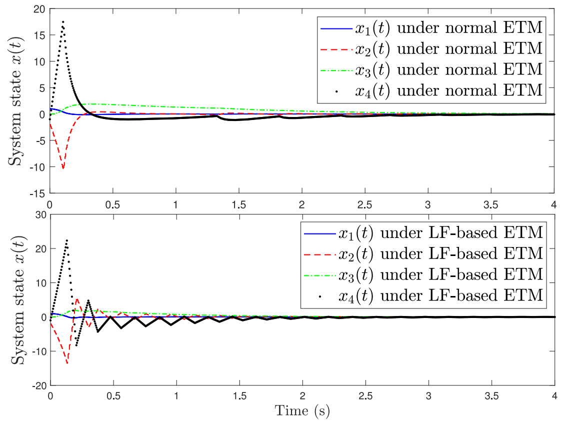

Fig. 1. System state response $x(t)$.

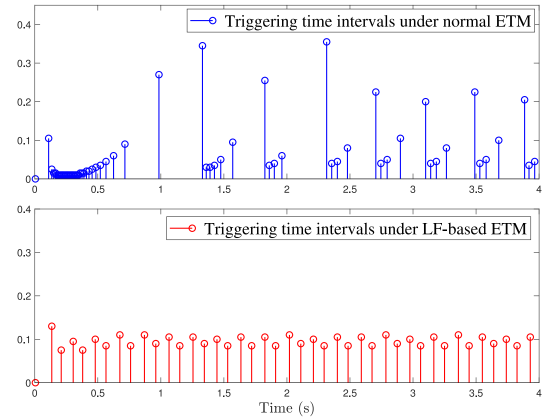

Fig. 2. Triggering time intervals.

TABLE I
THE NUMBER OF TRIGGERING TIMES: $\mathscr{N}$

Methods	Normal ETM (4)	LF-based ETM (6)
$\mathscr{N}$	196	44

V. EXAMPLE

Example 1: In this example, an inverted pendulum is simulated to illustrate the advantages of the developed strategies for the linear case. A linearized continuous-time state–space model of a pendulum [15] is considered as:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 63.25 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -33.31 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ -520.72 \\ 0 \\ 804.13 \end{bmatrix} u(t).$$

By discretizing the above system with sampling period 0.005s, the discrete-time system parameters are obtained as:

$$A = \begin{bmatrix} 1.0008 & 0.005 & 0 & 0 \\ 0.3163 & 1.0008 & 0 & 0 \\ 0.0004 & 0 & 1 & 0.005 \\ -0.1666 & -0.0004 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} -0.0065 \\ -2.6043 \\ 0.0101 \\ 4.0210 \end{bmatrix}.$$

A stabilization state feedback controller K, the matrices P and Q are chosen as:

$$P = 10^5 \times \begin{bmatrix} 1.3159 & 0.1998 & 0.1007 & 0.0986 \\ 0.1998 & 0.0311 & 0.0155 & 0.0156 \\ 0.1007 & 0.0155 & 0.0163 & 0.0080 \\ 0.0986 & 0.0156 & 0.0080 & 0.0081 \end{bmatrix},$$

$$Q = \begin{bmatrix} 6 & 5 & 3 & 4 \\ 5 & 6 & 3 & 4 \\ 3 & 3 & 6 & 2 \\ 4 & 4 & 2 & 6 \end{bmatrix}, \ K = \begin{bmatrix} 0.3662 \\ 0.0539 \\ 0.0034 \\ 0.0082 \end{bmatrix}^\top.$$

In simulation, we choose $x(0) = [1 \ \ -2 \ \ 0 \ \ -1]^\top$ and $\delta = 0.01$, $\theta = 0.25$. The comparison curves of state responses under the normal ETM (35) and LF-based ETM (36) are drawn in Fig. 1. The corresponding triggering instants and intervals are given in Fig. 2 and Table I. Fig. 3 illustrates the evolutions of LF $V(x(k))$ under the two ETMs.

It is seen form Fig. 1 that the system are well stabilized by the chosen controller and the considered two ETMs. In terms of Fig. 2 and Table I, the number of triggering events under the normal ETM is 196, and the one under our LF-based ETM is 44. Namely, one can observe that the proposed LF-based ETM dramatically reduce the number of triggering events and generate a larger average inter-event time than the normal ETM. Moreover, from Fig. 3, it is noticeable that the considered LF can have some small increases under the LF-based ETM, while it is strictly monotonic decreasing for the normal ETM.

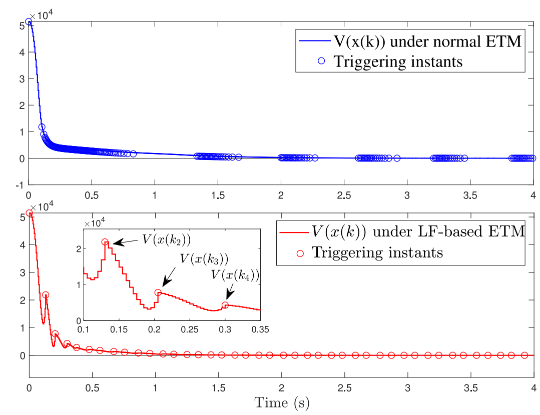

Fig. 3. Triggering time intervals.

Example 2: In this section, a nonlinear system is employed to show the validity of the proposed LF-based ETM. An event-triggered control nonlinear discrete-time system with the following form [13] is considered as:

$$\begin{aligned} x(k+1) &= x(k) + 0.05|x(k)| + 0.2u(k) \\ u(k) &= -x(k_i), \quad k \in \Omega_i. \end{aligned} \tag{37}$$

Then, the closed-loop system is obtained as:

$$x(k+1) = 0.8x(k) + 0.05|x(k)| - 0.2\epsilon(k). \tag{38}$$

We choose an ISS-LF $V(x(k)) = x^\top(k)x(k)$ for system (37). Then, the corresponding normal ETM and the LF-based ETM are obtained as

$$k_{i+1} = \inf_{k \in \mathbb{N}} \{k > k_i | s(x(k), \epsilon(k)) > 0\}, \tag{39}$$

$$k_{i+1} = \inf_{k \in \mathbb{N}} \{k > k_i | s(x(k), \epsilon(k)) + \theta \sum_{\kappa=k_i}^{k} \Delta V(x(\kappa)) > 0\}, \tag{40}$$

respectively, where $s(x(k), \epsilon(k)) = -0.11\delta\|x(k)\| + 0.21\|\epsilon(k)\|$.

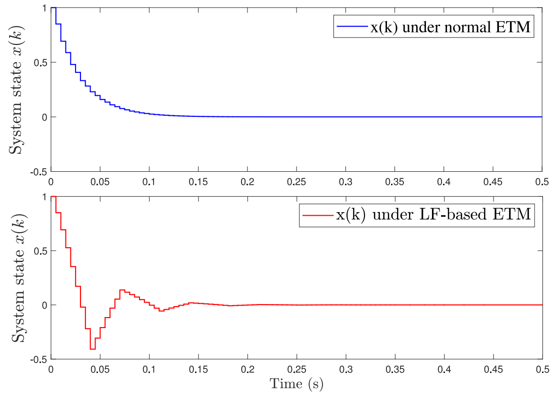

Fig. 4. System state response $x(t)$.

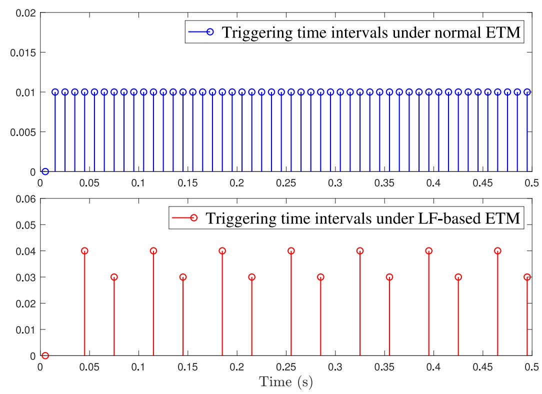

Fig. 5. Triggering time intervals.

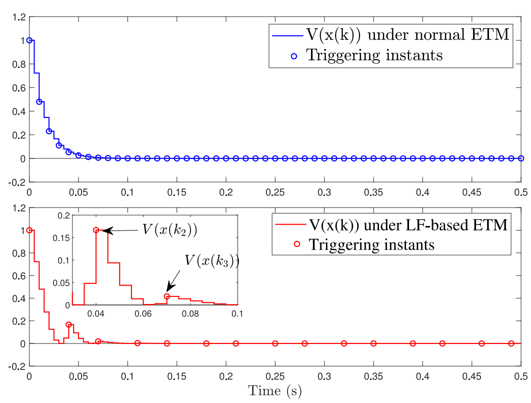

Fig. 6. Triggering time intervals.

TABLE II
THE NUMBER OF TRIGGERING TIMES: $\mathscr{N}$

Methods	Normal ETM (4)	LF-based ETM (6)
$\mathscr{N}$	50	15

For simulation, we choose the sampling step $0.005s$, $x(0) = 1$, $\delta = 0.1$ and $\theta = 0.4$. The curves of state responses under different ETMs are compared in Figure 4. The comparisons of triggering instants and LF are shown in Fig. 5, Table II and Fig. 6.

According to the above figures, it is observed that the asymptotic stability of the closed-loop system (38) is stabilized with the controller $u(k)$ under the normal ETM (4) and the LF-based ETM (6). According to Fig. 5 and Table II, it is noted that the numbers of triggering events of two ETMs are 50 and 15. This implies that the packet transmission rate can be significantly reduced from 50% under the normal ETM (4) to 15% under the proposed LF-based ETM (6).

VI. CONCLUSION

This brief investigates the event-triggered control for nonlinear discrete-time CPSs via a new LF-based ETM. To relax the normal ETM under the premise of maintaining system stability, the sum of LF difference between two consecutive events is used to design the triggering condition. In contrast to the requirement of a decrescent LF under the normal ETM, some increases of the considered LF is allowable to guarantee the asymptotic stability under the developed LF-based ETM. Therefore, the communication load can be reduced dramatically and more network resources are saved. Finally, two examples are carried out to demonstrate the benefits of the presented scheme. Note that the controller gain is given in advance, which can not be co-designed with the triggering conditions. Thus, the co-design issue of controller gain and the proposed LF-based ETM deserves more investigations in the future.

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